Budget uncertainty in a monetary union

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Abstract

Governments in a monetary union fear spending disturbances. We distinguish them according to their ability to hedge against these disturbances and assume that they derive their optimal fiscal decisions by using a robust control approach. Results show that governments being highly vulnerable to spending disturbances set excessive tax rates, thereby exacerbating the fiscal pressure detrimental to output and obliging the central bank to conduct an expansionary monetary policy. Countries whose governments have higher ability to hedge against spending disturbances then suffer from the inflationary consequences of this monetary policy.
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1 Introduction

Uncertainty about key macroeconomic aggregates and relationships governing the economy is an important challenge for policymakers and substantially influences their choices.[1] In recent years, uncertainty seems to have become particularly challenging for fiscal policy-makers, and this, especially in the European Monetary Union (EMU) member countries. Fiscal authorities may face uncertainty stemming from lots of factors, including bond markets

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[1] A series of papers has examined the monetary policymakers' decisions in the face of uncertainty about the structural parameters of the economy. Among them, Peersman and Smets (1999), Giannoni (2002), Söderström (2002), Gros and Hefeker (2002) or Tillmann (2009a) for instance confirm Brainard's (1967) classical result which is to say that monetary authorities tend to act more carefully if they are confronted to some model uncertainty. Other papers like Stock (1999) and Onatski and Stock (2002) challenge this view and find that uncertainty may lead to more vigorous interest rate setting.

pressures as well as factors due to economic, social or political instability. The analysis of the implications of the fiscal policy-makers' uncertainty on macroeconomic outcomes thus represents a crucial topic to be put into the agenda of academics and practitioners. As far as we know, only two theoretical studies have dealt with this issue. Di Bartolomeo *et al.* (2009) and Di Bartolomeo and Giuli (2011) develop a model where monetary and fiscal authorities face uncertainty about the parameters describing their policy effectiveness. They qualify Dixit and Lambertini (2003)'s result obtained under the *symbiosis assumption* by showing that, under multiplicative uncertainty, even if monetary and fiscal authorities share identical targets for output and inflation, the achievement of these targets is no longer guaranteed.

In this paper, we consider a monetary union (MU) where national governments face uncertainty about the exact amount of public spending to be financed. More precisely, when setting their fiscal policy, governments fear some spending disturbances they do not necessarily have under control and which may alter or invalidate their budget projections. In addition, we assume that their spending can only be financed by taxation.[2] To model their fiscal decision-making under this type of uncertainty, we use the *robust control* approach.[3] Through this approach, we assume that governments are unable to define any probability distribution to spending disturbances. To set an optimal tax rate under these circumstances, they seek to select a fiscal policy that is robust to the most pessimistic perspective, i.e. that remains optimal even under the worst possible outcome of spending disturbances.

We allow for some asymmetry among the MU governments in the sense that we consider that some of them have lower ability to hedge against spending shocks and are therefore more vulnerable to budget uncertainty than others. This vulnerability can be so high that they are obliged to attach priority attention to spending stabilisation. Their fiscal policy is then constrained as it mainly focused on short-term spending imperatives at the expense of the economy's output performance. We refer to this type of governments as "fiscal-constrained".

Obviously, each national government's spending uncertainty affects its fiscal decisions but not only. It is also likely to interfere in the common central bank's decisions and thereby affect the MU partner countries' situation. In this paper, our objective is twofold. First, we seek to outline the consequences of the governments' spending uncertainty for their fiscal decisions and, through it, for macroeconomic outcomes in the MU member countries. Second, we study the implications of this uncertainty for the macroeconomic

[2]This is consistent with the EMU situation where governments face a balanced-budget requirement in the medium term.

[3]The robust control approach has been introduced into economic models by Hansen *et al.* (1999) and Hansen and Sargent (2005, 2008). A number of recent papers has used this approach to determine the optimal monetary policy in the case where some uncertainty is faced by central bankers (see for instance, Giannoni, 2002, 2007; Tillmann, 2009a,b, 2014, 2019, Woodford, 2010 among others).

effects of monetary institutions. In particular, we examine how the central bank's degree of inflation aversion as well as the number of MU member countries affect macroeconomic performances – in terms of inflation and output – in the presence of budget uncertainty.

It appears from our analysis that when the MU governments are vulnerable to spending disturbances, they set high tax rates. This exacerbated fiscal pressure translates into low output in the member countries, obliging the central bank to be accommodating by conducting an inflationary monetary policy. In an asymmetric MU, this result implies that countries where the government has high ability to hedge against spending disturbances will rapidly feel inconvenienced by the too inflationary monetary policy set by the central bank in reaction to the bad output situation in the partner countries whose government has lower ability to hedge against spending disturbances.

Our analysis also examines how changes in the monetary institutional environment can affect the governments' fear of spending disturbances. Results reveal that an increase in the central bank's concern about inflation exacerbates this fear, leading fiscal-constrained governments to set higher taxes at the expense of output. If these governments are in the majority in the MU, the central bank is induced to be more accommodating by conducting an inflationary monetary policy. Therefore, quite counter-intuitively, an increase in the central bank's concern about inflation can result into higher inflation. By studying the macroeconomic effects of a MU enlargement, we show that the entry of new countries aggravates the heterogeneity of the current members' output situation. Moreover, if the new entrants suffer from greater budget uncertainty than the current members, the latter may also experience an increase in the MU-inflation rate.

The rest of the paper is organized as follows. In the next section, we develop a MU-model with monetary-fiscal strategic interactions in which we integrate the issue of budget uncertainty. The implications of this uncertainty for the member countries' economic outcomes are discussed in section 3. Section 4 presents the macroeconomic effects of changes in the monetary institutional environment in the presence of governments' budget uncertainty. A final section concludes.

2 The model

We consider a monetary union (MU) composed of n countries (indexed by i, $\forall i = 1, ..., n$). Monetary policy is set by a common central bank (CB) whereas fiscal policies are decided at the national level by the member countries' governments.

Output x_i in country i is given by:

$$x_i = \pi - \pi^e - \tau_i \tag{1}$$

where π and π^e are the actual and expected inflation rates respectively; τ_i

defines the tax rate in country i. Behind this relation lies the idea that unexpected inflation, by eroding real wages, induces firms to augment their demand for labour and thus their production. Greater taxation on the firms' revenues, on the contrary, discourages production. Hence, as is common in this literature (see Beetsma and Bovenberg, 1998, 1999 among others), fiscal policy has a negative impact on aggregate supply via taxation. We thus assume that taxation is used for consumption only and not investment, so that it has no productive effect on supply.[4]

The common CB cares about deviations of both, inflation and output from their respective targets which, for convenience, we assume to be equal to zero. Its loss function is given by:

$$L^{CB} = I\pi^2 + x^2 \tag{2}$$

where π and $x = \sum x_i/n$ respectively define the MU-wide inflation rate and the average output level in the monetary union. We here suppose that the central bank perfectly controls the MU-wide inflation rate so that its monetary policy instrument is π. Parameter I measures the CB's relative concern for inflation with respect to output. It can also be interpreted as the CB's degree of inflation aversion.[5]

National governments are concerned about both, the level of output and of public spending in their economy. The objectives of government in country i (henceforth, government i) are summarized as follows:

$$L_i^G = x_i^2 + \alpha \left(g_i - \tilde{g}_i\right)^2 \tag{3}$$

where g_i and $\tilde{g}_i$ respectively define the country i's actual and targeted levels of public expenditures as shares of output. Parameter α measures the relative importance government i gives to its spending objective.[6]

When setting its fiscal decision, government i faces the following budget constraint:

$$g_i = \tau_i \tag{4}$$

This equation corresponds to a balanced budget requirement where taxation is the only source of financing public expenditures.

Government i will thus have to tolerate some tax distortions in order to finance its positive target of public expenditures $\tilde{g}_i$. The type of expenditures

[4]Yet, the variable τ_i could also be interpreted as the tax rate net of public investment so as to capture the productivity enhancing role of public expenditures. See for instance Ismihan and Ozkan (2004) who explicitly consider the composition of public spending by distinguishing between public sector consumption and investment.

[5]The literature also refers to I as the CB's degree of independence. For a distinction between CB inflation aversion (or conservatism) and independence, see for instance Eijffinger and Hoeberichts (1998, 2008), Hughes Hallett and Weymark (2005), Weymark (2007) and Hefeker and Zimmer (2011).

[6]For convenience, we assume that national governments share the same weight α as well as the same output target, which is normalized at zero.

that we consider here corresponds to public consumption such as salaries of public employees and other current spending generated by the supply of public goods. It also encompasses social security spending and the payment of interest on public debt (which we do not explicitly model). We here abstract from the beneficial impact public spending may have on output as g_i does not integrate public investment goods.

As it is noticeable, the spending target $\tilde{g}_i$ is specific to each country. For government i, it writes:

$$\tilde{g}_i = \overline{g} + \epsilon_i \tag{5}$$

where $\overline{g}$ represents a fixed level of public spending and is expressed as a share of output. It can be interpreted as an average socially optimal level of spending in the MU.

According to this expression, government i has its own spending target that may deviate from the average socially optimal level in the MU $\overline{g}$. Indeed, government i's spending can be affected by lots of specific disturbances such as bad business cycles developments, political and social instability – obliging the government to increase social expenditures – or financial market frictions putting pressure on public debt interest rates. These disturbances are captured by ϵ_i and render the exact amount of its spending highly uncertain for the government. To hedge against this uncertainty, government i may want to keep some leeway in its decision-making by taking account of ϵ_i in its spending target.

A crucial assumption here is that government i is unable to assign any probability distribution over alternative outcomes of ϵ_i. To determine its fiscal policy under this assumption, we use a robust control (non-Bayesian) approach which consists in setting its tax rate (and thus spending level) so that it is robust to the worst possible realization of ϵ_i.

Following Hansen and Sargent (2005, 2008), this approach can be modeled as a game between government i and a fictitious "evil agent" whose aim is to set the disturbances ϵ_i so as to maximize the government's welfare loss.

Concretely, this implies for government i to solve the following *minmax* program:

$$\min_{\tau_i} \max_{\epsilon_i} L^G = x_i^2 + \alpha \left(g_i - \tilde{g}\right)^2 - \theta_i \epsilon_i^2 \tag{6}$$

where θ_i defines the government's ability to hedge against spending disturbances. This parameter varies in the interval $[\underline{\theta}; \infty[$.[7] The certainty case corresponds to $\theta_i \to \infty$. This is the common case in the literature where it is assumed that governments do not suffer from spending uncertainty and thus have a perfect control over their budget. Parameter θ_i is indexed by i, implying that we allow for some asymmetry among MU-member countries.

An alternative method to model fiscal uncertainty would be to use a Bayesian approach where spending disturbances correspond to white noise

[7] The lower bound $\underline{\theta}$ will be defined below in footnote 10.

stochastic shocks and where fiscal uncertainty is given by the variance of these shocks. This method suggests that governments can attach priors to budget disturbances, which seems difficult in reality. In particular, in a context of high political, social or financial instability it appears reasonable to assume that governments are unable to statistically measure their budget uncertainty.

The timing of the game is as follows. First, in each member country, the private sector rationally determines inflation expectations through the nominal wage setting process. Then, simultaneously, the MU governments determine their tax rate τ_i using the robust control approach detailed above. Finally the common central bank selects π, the MU inflation rate. Since in practice monetary policy can be adjusted more quickly than fiscal policy, we assume that, when taking their decisions, governments anticipate the central bank's reaction to their decisions and thus act as Stackelberg leaders.[8]

The game is solved by backward induction and we begin by considering the common central bank's program.[9] Minimizing loss function (2) with respect to π and taking the member countries' aggregate supply functions (1) as given, we have:

$$\pi = \frac{\pi^e + \tau}{1 + I} \tag{7}$$

where $\tau = \sum_i \tau_i / n$ defines the average MU-wide tax rate.

With the rational expectations assumption ($\pi^e = \pi$), we obtain:

$$\pi = \frac{\tau}{I} \tag{8}$$

Solving government i's program with the robust control approach and taking rational expectations (so that, $\pi^e = \pi$), we obtain the equilibrium level of tax rate in country i:

$$\tau_i = \frac{\alpha \overline{g} \theta_i \left(1 + I\right) n}{\theta_i \left[n \left(1 + I\right) \left(1 + \alpha\right) - 1\right] - \alpha \left[n(1 + I) - 1\right]} \tag{9}$$

Integrating this expression into the central bank's reaction (8), the output function (1) as well as the budget constraint (4) and considering rational expectations, we have the equilibrium values for output and public expenditures in country i, respectively: $x_i = -\tau_i$ and $g_i = \tau_i$, and also obtain

[8] An interesting alternative, could be to consider a reversed timing where the central bank is the Stackelberg leader and the national governments, the followers. This sequence of events could be relevant to describe particular cases such as the euro crisis where the European Central Bank often argued to "buy time" for national governments. We thank an anonymous referee for having raised this idea. Calculations for this timing are available on request.

[9] Details on the derivation of the equilibrium outcomes are provided in the appendix.

the equilibrium expression for inflation in the MU: $\pi = \tau$ (with, as already specified, $\tau = \sum_i \tau_i/n$).[10]

We observe that macroeconomic performances in country i – i.e. τ_i, x_i and g_i – only depend on θ_i, the government i's ability to hedge against spending disturbances, and not on θ_j ($\forall j \neq i$), the other governments' uncertainty parameter. The MU-wide inflation rate $\pi =$ is the only variable that depends on the θ parameter of all national governments.

Moreover, in our model, the distortion from the first best outcome is due to the presence of a positive (fixed) spending target $\overline{g} > 0$, obliging the government to collect a positive amount of taxes ($\tau_i > 0$).[11] This in turn reduces output ($x_i < 0$) and forces the central bank to implement an expansionary and thus inflationary monetary policy ($\pi > 0$).

As for country i's equilibrium spending gap ($g_i - \overline{g}$), it can either be positive or negative, depending on the value of θ_i and α. For instance, when $\alpha > \theta_i$, government i pays high attention to its spending level (α high) whereas its ability to hedge against spending disturbances is relatively low (θ_i low). $\alpha > \theta_i$ also implies that $\alpha/\theta_i > 1$, which means that government i is more preoccupied by its spending disturbances (given the high importance it attaches to spending and its low ability to hedge against the disturbances) then by the economy's output level.[12] In this case, government i does not hesitate to set strong fiscal pressure in order to finance excessive expenditures even though this discourages the private sector's productive investments. Here, country i's spending gap appears to be positive. In what follows, we qualify this type of government as *fiscal-constrained*, prioritizing expenditures imperatives at the expense of output. $\alpha/\theta_i > 1$ can thus be interpreted as the degree of government i's fiscal-constraint ; the higher this ratio, the higher the government's constraint for short-term spending imperatives.

On the basis of these results, we can investigate the macroeconomic consequences of the governments' budget uncertainty.

Budget uncertainty is defined as the governments' incapacity to perfectly control their spending. Formally, budget uncertainty is inversely related to θ_i: the lower θ_i, the lower the government i's ability to hedge against spending disturbances.

From expression (9), we observe that budget uncertainty exacerbates the tax distortion due to the positive spending target.[13] We obtain the following

[10]We assume that, for each government, θ_i is sufficiently high for the denominator of expression 9 to be positive. This implies: $\theta_i > \frac{\alpha[n(1+I)-1]}{n(1+I)(1+\alpha)-1} = \underline{\theta}$; $\underline{\theta}$ being the minimum value of θ in our model.

[11]The first best outcome corresponds to a situation where: $\tau_i = x_i = \pi = 0$ and $g_i = \overline{g}$.

[12]Indeed, according to the government's loss function (Eq. 6), the relative weight attached to output is equal to 1.

[13]Note that budget uncertainty disappears when $\theta_i \to \infty$, which means that government i has a perfect control over its spending. By setting $\theta_i \to \infty$, equilibrium outcomes

result.

Result 1. *A decrease in θ_i, the government i's ability to hedge against spending disturbances translates into:*
i) higher levels of taxation and public spending and into a lower level of output in country i.
ii) an increase in the MU-wide inflation rate π – proportionally to $1/n$ corresponding to the country's weight in the CB's decision process.

Proof: See appendix.

Indeed, when government i has low control over its spending (θ_i low), it fears large disturbances and feels obliged to set high taxes in order to collect enough fiscal revenues to compensate for these disturbances. As a consequence to this strong fiscal pressure, output in country i is depressed. This in turn induces the central bank to implement a more expansionary and thus inflationary monetary policy, depending on the country i's weight in its decision process ($1/n$).

MU countries whose government has higher control over its spending (θ_j high, $\forall j \neq i$) then suffer from the inflationary consequences of the CB's accommodating policy.

3 Macroeconomic effects of changes in monetary institutional parameters

After having clarified the implications of the governments' spending uncertainty, we can now proceed to examine the macroeconomic effects of a change in the CB's degree of inflation aversion as well as in the number of MU member countries.

Result 2. *An increase in the central bank's degree of inflation aversion I translates into :*
i) higher taxes τ_i and public spending g_i as well as into lower output x_i in the member countries where the government is fiscal-constrained (i.e. is characterized by $\frac{\alpha}{\theta_i} > 1$).
ii) higher inflation in the MU if the majority of member countries are characterized by a relatively low θ_i (i.e. $\theta_i < \theta_1$).

Proof: See appendix.

correspond to those observed in the standard case (see for instance Hefeker and Zimmer (2011) where a similar model is used with the assumption $\theta_i \to \infty$).

This results states that when the member countries' governments are fiscal-constrained ($\frac{\alpha}{\theta_i} > 1$), the CB's degree of inflation aversion exerts a detrimental impact on their macroeconomic performances.

Clearly, if the CB is highly inflation averse, it accommodates national tax hikes to a lesser extent, thereby rendering them more costly in terms of reduced output. Two opposite fiscal reactions to high CB inflation aversion are observable, depending on the governments' concern for spending disturbances relatively to their concern for output.

In the standard case where governments do not suffer from spending uncertainty ($\theta_i \to \infty$), they adapt to high CB inflation aversion by setting low taxes. This improves their country's output performance but it also contributes to exacerbate their spending gap. Indeed, as has been observed earlier, in the case where $\alpha/\theta_i < 1$ (and thus where $\theta_i \to \infty$), the equilibrium spending level g_i is lower than the target $\overline{g}$, so that the deviation $(g_i - \overline{g})$ is negative. By reducing the tax level, and thereby the spending level g_i, high CB inflation aversion finally aggravates this deviation.

However, in the case where governments are exposed to budget uncertainty (θ_i has a finite value), high CB aversion to inflation makes them feel more vulnerable to spending disturbances.[14] This in turn leads them to set higher taxes.

When governments are fiscal-constrained ($\frac{\alpha}{\theta_i} > 1$), the *fear of spending disturbances*-effect prevails, explaining their high tax rate in response to high CB inflation aversion and consequently, their low output level and high spending gap $(g_i - \overline{g})$.

If these member countries are in the majority, the overall impact of CB inflation aversion on the MU-wide tax level is positive, translating into an expansionary monetary policy (see Eq. (8)). In the case where the MU-governments' θ_i is very low, this effect may be so strong that it compensates the direct inflation-reducing effect of CB inflation aversion highlighted in the standard literature on CB design. We then observe the counter-intuitive result where high CB inflation aversion is associated with high inflation.

If this is the case, the CB's concern for inflation eventually proves detrimental for real economic performances and this, not only in member countries where the government is fiscal-constrained but also in the others. Indeed, even though CB inflation aversion has a beneficial impact on production in these countries, it contributes to exacerbate spending deviations and inflation.

Result 3 examines the implications of budget uncertainty for the macroeconomic effects of a MU enlargement.

[14] This latter effect hinges on the observation that a higher degree of CB inflation aversion triggers an increase in ϵ_i, the government i's anticipation of spending disturbances. Mathematically, this writes as follows: $\frac{\partial \epsilon_i}{\partial I}$. Calculations are available upon request.

Result 3. *An increase in the number of monetary union member countries: i) leads to higher taxes τ_i and public spending g_i as well as to lower output x_i in countries with a fiscal-constrained government ($\frac{\alpha}{\theta_i} > 1$). ii) translates into an increase in inflation for countries whose government is less vulnerable to spending disturbances (θ_i higher) than the government of the new entrants.*

Proof: See appendix.

The process of MU enlargement triggers similar macroeconomic effects than an increase in CB inflation aversion. Indeed, as can be observed from expression (8), both institutional changes (increase in n and in I) contribute to moderate the CB's accommodating response to national tax decisions. This has two implications: first, it aggravates the output-reducing impact of tax hikes and second, it exacerbates the governments' fear of spending disturbances.[15]

How governments adapt to this situation depends on whether they are more concerned by output or by their spending and spending disturbances (i.e whether α/θ_i is higher or lower than 1).

Fiscal-contrained governments with $\frac{\alpha}{\theta_i} > 1$ will prefer to raise taxes and spending even though this implies a deterioration of output. MU enlargement finally results into poorer output performance in their countries.

As for member countries whose government is not fiscal-constrained (with $\frac{\alpha}{\theta_i} < 1$), even if they observe an improvement in their output performance with the MU enlargement – this effect has already been stressed by Beetsma and Bovenberg (1998) – they may experience an increase in inflation due to the more accommodating policy the CB adopts in response to their new partner countries' higher fiscal pressure.

4 Conclusion

In this paper, we extend a simple MU-model with strategic monetary-fiscal interactions by considering the case where national governments suffer from budget uncertainty due to spending disturbances. An important element in our analysis is the governments' ability to hedge against these disturbances. Our results reveal that governments whose ability is low feel obliged to set high taxes, thereby deteriorating output in their country. Their decisions however also have economic repercussions in countries whose government is less vulnerable to spending disturbances as the CB then conducts a more accommodating monetary policy resulting in higher inflation in the MU.

[15]This latter effect hinges on the observation that a higher number of member countries triggers an increase in ϵ_i, the government i's anticipation of spending disturbances. Mathematically, this writes as follows: $\frac{\partial \epsilon_i}{\partial n}$. Calculations are available upon request.

It also appears from our analysis that the governments' fear of spending disturbances depends on the monetary institutional environment. Indeed, we show that this fear is exacerbated when the central bank has strong concern for inflation. Fiscal-constrained governments, being focused on short-term spending imperatives, thus react to central bank inflation aversion by increasing taxes even though this implies lower output. As a result, the central bank is tempted to accommodate these decisions with an expansionary and thus inflationary monetary policy. If budget uncertainty in the MU is very high, this mechanism may be so strong that it challenges the traditional result that CB inflation aversion contributes to lower inflation.

Moreover, by allowing for asymmetric vulnerability of national governments to budget uncertainty, we observe that the MU enlargement is likely to aggravate the heterogeneity of the member countries' economic situation: the most-vulnerable countries may experiment a decrease in their already-low output level whereas the least-vulnerable countries may see their output level increase. Besides, it is quite possible that member countries whose government has high control over spending disturbances will have to accept higher inflation in the MU. This is the case if the government of the new entrants is extremely vulnerable to spending disturbances, obliging the common central bank to set a very accommodating and thus inflationary policy.

This paper takes into account the existence of some budget uncertainty that can weaken national governments in a MU and proposes a modelling of this uncertainty. Its results open the discussion to the broader debate on the economic governance in a MU. How should monetary and fiscal institutions be designed for the optimal functioning of a MU in the presence of heterogeneous budget uncertainty of national governments? This question may be of interest for future research.

Appendix

Derivation of the equilibrium tax rate τ_i:

Equilibrium outcomes are derived from a three-step procedure. Solving the game backwards:

1. We begin by minimizing the central bank's loss function (2) with respect to π and taking the member countries' aggregate supply functions (1) as given. We obtain:

$$\pi = \frac{\pi^e + \tau}{1 + I} \tag{10}$$

where $\tau = \sum_{i=L,H} \tau_i / n$.

2. We then solve government i's ($i = L, H$) minmax program.

In minimizing government i's loss function (6) with respect to τ_i, we obtain the following condition:

$$[n\,(1+I)\,(1+\alpha)-1]\,\tau_i - [\alpha n\,(1+I)]\,\epsilon_i = \alpha n\,(1+I)\,\overline{g} + [n\,(1+I)-1]\,(\pi - \pi^e) \tag{11}$$

In maximizing government i's loss function (6) with respect to ϵ_i, we obtain the following condition:

$$\alpha\tau_i + (\theta_i - \alpha)\epsilon_i = \alpha\overline{g} \tag{12}$$

Combining both conditions leads to the following expression for government i's tax rate:

$$\tau_i = \frac{\alpha\theta_i\,(1+I)\,n\overline{g} + (\theta_i-\alpha)\,[n(1+I)-1]\,(\pi-\pi^e)}{\theta_i\,[n\,(1+I)\,(1+\alpha)-1] - \alpha\,[n(1+I)-1]} \tag{13}$$

3. Taking rational expectations ($\pi^e = \pi$), we finally obtain the equilibrium tax rate τ_i as given in equation (9).

Proof of Result 1:

i) Differentiating τ_i, the equilibrium tax rate in country i, with respect to θ_i leads to:

$$\frac{\delta\tau_i}{\delta\theta_i} = \frac{-\overline{g}\alpha^2 n(1+I)[n(1+I)-1]}{\{\theta_i\,[n\,(1+I)\,(1+\alpha)-1] - \alpha[n(1+I)-1]\}^2} < 0 \tag{14}$$

Consequently, we have: $\frac{\delta g_i}{\delta\theta_i} = \frac{\delta\tau_i}{\delta\theta_i} < 0$ and $\frac{\delta x_i}{\delta\theta_i} = -\frac{\delta\tau_i}{\delta\theta_i} > 0$.

ii) By differentiating π, the equilibrium inflation rate in the MU, with respect to θ_i, we obtain: $\frac{\delta\pi}{\delta\theta_i} = \frac{1}{n}\frac{\delta\tau_i}{\delta\theta_i} < 0$.

Proof of Result 2:

i) Differentiating τ_i with respect to I leads to:
$\frac{\delta\tau_i}{\delta I} = \frac{\alpha n\tilde{g}\theta_i(\alpha-\theta_i)}{\{\theta_i[n(1+I)(1+\alpha)-1]-\alpha[n(1+I)-1]\}^2}$. This derivative is positive when $\alpha > \theta_i$.
As a result: $\frac{\delta g_i}{\delta I} = \frac{\delta\tau_i}{\delta I}$ and $\frac{\delta x_i}{\delta I} = -\frac{\delta\tau_i}{\delta I}$ are respectively positive and negative when $\alpha > \theta_i$.

ii) According to expression (8), we have: $\frac{\delta\pi}{\delta I} = \frac{\sum_i}{n}\frac{\partial(\tau_i/I)}{\partial I}$.
Differentiating $\frac{\tau_i}{I}$ with respect to I yields:
$\frac{\partial(\tau_i/I)}{\partial I} = \frac{\alpha n\theta_i\tilde{g}\left\{-\theta_i\left[n(1+I)^2(1+\alpha)-1\right]+\alpha\left[n(1+I)^2-1\right]\right\}}{\{I\theta_i[n(1+I)(1+\alpha)-1]-\alpha I[n(1+I)-1]\}^2}$. This derivative is positive for $\theta_i < \frac{\alpha\left[n(1+I)^2-1\right]}{n(1+I)^2(1+\alpha)-1} \equiv \theta_1$.[16]

[16]Note that $\theta_1 > \underline{\theta}$; $\underline{\theta}$ being the minimum value of θ_i defined in footnote 10.

Hence, if for the majority of member countries $\theta_i < \theta_1$, then $\frac{\sum_i}{n}\frac{\partial(\tau_i/I)}{\partial I}$ is positive, implying that $\frac{\delta\pi}{\delta I} >$ is positive too.

Proof of Result 3:

i) The macroeconomic effects of the monetary unification process are captured by considering the transition from the case where $n = 1$ to the case $n > 1$. These effects can thus be determined by differentiating the equilibrium expression of τ_i, g_i and x_i with respect to n. This leads to:
$\frac{\delta\tau_i}{\delta n} = \frac{\alpha\overline{g}\theta_i(1+I)(\alpha-\theta_i)}{\{\theta_i[n(1+I)(1+\alpha)-1]-\alpha[n(1+I)-1]\}^2}$. This derivative becomes positive when: $\alpha > \theta_i \Rightarrow \frac{\alpha}{\theta_i} > 1$. Consequently, $\frac{\delta g_i}{\delta n} = \frac{\delta\tau_i}{\delta n}$ and $\frac{\delta x_i}{\delta n} = -\frac{\delta\tau_i}{\delta n}$ become respectively positive and negative for $\frac{\alpha}{\theta_i} > 1$.

ii) We know from result 1 that countries with a high θ_i display a low level of taxation. If they join a MU with partner-countries where θ_i is relatively lower and thus taxation relatively higher, they may have to accept a more inflationary monetary policy as this latter is then set by a common CB on the basis of the average tax rate in the MU (see Eq.(8)).

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